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## GAUGING EPITOMIZED.

 O R, A
## Short Treatife of Gauging,

In which that Branch is rendered familiar to the Meanest Capacity.
TO WHICH ARE ADDED,

Accurate Tables for finding the Mean-Diameters and conle, ió of Cafks by Infpection.
A L S O,

A comprehenfive Ullage Table, and an accurate Method of Ullaging Cafks, by an eafy Rule adapted to it.

The whole illuftrated with proper Rules and Examples.

## By BENJAMIN WORKMAN,A.M.

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P \quad H I L A B C D L P A I A:
$$

Printed and fold by W. Young, the corner of Chefrut and Second-freets.

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## PR E FA C E.

THE original intention of the following Gauging-Tables, was to furnish gangers, merchants, and all others, to whom a knowledge of the contents of velgels was neceffary, with an expeditious, and at the fame time a more accurate method of gauging than any heretofore practied; and it is prefumed that fuccefs has refulted from the intention.

By the affiftance of this little Book, without any preceptor, any perfon acquainted with the first rules of arithmetic may learn to gauge, and become adroit in that branch.-Wine-merchants, grocers, clerks, \&c. will be enabled thereby to gauge their own liquors more accurately than any gauger could have done by their erronfous methods by the fide-rule.

Although the firft part was defined, only as a fort Sketch of gauging, yet it will

## $P R E F A G E$.

will be found to comprehend all the rules neceffary, or occuring, in the practice of the art; and befides form a bandfome compendium for fchools.

In this little Treatife there is introducoed a nero method of extracting the cube root more eafy than any that 1 have Seen.

The decimals, extractions of the Square and cube roots, geometrical definitions and principles, \&c. will be found extremely useful as an introduction to other important branches of the mathematics; as they are expreffed in clear and general terms.

## B. Workman.

## Philadelphia, ? <br> May 20th, 1788.$\}$

## TREATISE or GAUGING.

## -

## Сна f . I. <br> Of DECIMAL FRACTIONS.

## Defnitions.

1. Araction is a part or parts of unity, or any one whole thing which may be divided.
2. A fraction is generally expreffed by two numbers placed the one above the other, with a line drawn between them.
3. The number above the line is called the $N u$ merator, and the number below the line the Denominator. Thus, $\frac{1}{2}, \frac{2}{7}$, and $\frac{5}{5}$, are fractions, which are read, one-half, two-thirds, and five-tenths ; the numbers $\mathrm{I}, 2$, and 5 , are the numerators, and 2 , 3 , and 10 , the denominators.
4. Fractions which have 10, 100, 1000, E'c. (to wit, unit with a cypher or cyphers), for denominators, are called Decinal Fractions; thus, $\frac{5}{T 0}$ $\frac{33}{50} \frac{30}{305}, \frac{365}{1000}$, are decimal fractions: but all others are called Vulgar Fractions; fo $\frac{3}{5}, \frac{2}{3}$, and $\frac{25}{25}$, are vulgar fractions.
5. The denominator of a decimal fraction is never written down, for a point or comma being placed before the numerator fupplies its place;
thus, $\frac{5}{10}$ is written, 5 , alfo, 53 is the fame as $\frac{5.3}{706}$ and , 365 the fame as $\frac{305}{1000}$; moreover, $\frac{5}{100}$ is written, 05 .
6. In ofder to read a decimal fraction when written down, we are to conceive a cypher placed under every figure of it, and unit or 1 under the point, and this number will be the denominator; thus,, 5 is $\frac{5}{10},, 36$ is $\frac{36}{100}$, and , 032 is $\frac{32}{30} 0$, $E^{\circ} c$. Note, a decimal fraction which confifts of feveral figures nay be read after this manner, viz. by calling the figure next the point tenths, and the one next to this hundredths, the next thoufandths, $\mathcal{F}^{\circ} c$; thus, 365 may either be read threetenths, fix-hundredths, and five-thoufandths; or three hundred and fixty five thoufandths; for cither of thefe methods exprefs the fame thing.
7. A cypher or cyphers placed to the right hand of a decimal, never changes its value; thus, , 5 is the fame as, 50 or ,500: But every cypher placed to the left hand of a decimal, to wit, between the point and the fignificant figures, decreafes the value ten times; fo, 05 is ten times le's than ,5, and ,0035 is one hundred times lefs than $35, \mathcal{E}^{\circ} c$.
8. Becaufe decimal fractions increafe and decreafe in the very fame manner that whole numbers do ; that is, in a tenfold proportion ; therefore they are joined together like one number; only the point is to ftand before the decimal, in order to feparate it from the whole number; thus, 25,5 gallons, is 25 gallons, and five-tenths, (or one half of another gallon), $\sigma^{2} c$.
9. Every
10. Every operation in decimal fractions is performed exactly as if it were in whole numbers, due regard being had to the decimal point; for as ten units make ten, ten tens one hundred, ten hundreds one thoufand, $\mathcal{E}^{\circ} c$; fo, in like manner, ten tenths make unit or one, ten hundredths one tenth, ten thoufandths one hundredth, Ejc. And from hence it is manifeft, that any perfon who is acquainted with whole numbers, may, in one hours time, make himfelf mafter of decimals, becaufe the method of operation is exactly the fame in both, as was faid above.

## ADDITION of DECIMALS.

Rule.

Place the given numbers fo that the decimal points may ftand directly under one another; then add as in whole numbers, and place the decimal point in the fum under thofe above, and the work is done.

Examples.

|  |  |  |  | 0,5 |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1,36 | ,3 |
|  | 367,52 | 42,56 | 2,25 | ,6 |
|  | 213,78 | 8,7 | ,76 | ,03 |
|  | 256,51 | 92,3 | ,866. | ,8,9 |
|  | 526,25 | 51,245 | 1,6 | 1,0 |
|  | 120,3 | 2,1 | 2,03 | 2, |
| Sun, | 1484,36 | 196,905 | 8,866 |  |

## SUBTRACTION of DECIMALS.

## Rule.

Place the given numbers in fuch a manner that the decimal points may ftand directly under one another ; then fubtract as in whole numbers, and place the decimal point in the remainder under thofe above, and the work is done. Note, If the leffer number have more decimal figures than the greater, then you are to conceive as many cyphers annexed to the greater, fo as to make it bave as many decimal figures as the other.

Examples.



## MULTIPLICATION of DECIMALS.

## Rule.

Multiply exactly as in whole numbers, and from the product cut off as many figures for the decimal from the right hand towards the left, as there are decimal places in both multiplier and multiplicand; but if the product fhould not have as many figures as ought to be cut off, then the defect
defect muft be made up by prefixing as many cyphers to it as are neceffary.

| MultiplyBy | Examples. |  |  |
| :---: | :---: | :---: | :---: |
|  | 36,5 | 36,25 | 26,356 |
|  | 1,23 | 2,15 | 112 |
|  | 1095 | 18125 | 52712 |
|  | 730 | 3625 | 26356 |
|  | 365 | 7250 | 26356 |
| Product, | 44,895 | 77,9375 | 2951,872 |
| Multiply | ,2561 | ,2136 | ,32 276 |
| By | ; 5 | ,007 | , 1 , 08 |
| Product, | ,12801 | 0014952 | 032 22,08 |

## DIVISION of DECIMALS.

## Rule.

Divide exactly as in whole numbers, and from the right hand of the quotient, cut off as many figures for the-decimal, as is the excefs of the number of decimal places in the dividend above thofe of the divifor: But be careful to obferve, that you are always before you begin ta divide, to caufe your dividend to have at leaft four or five decimal figures more than the divifor, (if it have them not already), which is done by annexing cyphers to it. Moreover, if the quotient fhould not have as many figures as are to be cut off; then fufficient number of cyphers to it.

| Dividie 627,632615 by 25,25 ? |  |
| :---: | :---: |
| $25,25) 627,632615(24,8567$ quotient. |  |
| $5050 .$. |  |
| 12263 | Here becaufe there are fix de- |
| 1010 | cimal figures in the dividend, and two in the divifor; there- |
| 21632 |  |
| 20200 | fore the quotient is to have |
| 14326 | four decimal places; that is |
| 12625 | Note, the remainder is dropt, |
| 17011 | as being of little value; pro- |
| 15150 |  |
| 18615 | 5 number of decimal places |
| $176 \% 5$ | 5 already in the quotient. |
| 940 |  |
| Divide 2565 by 2,4? |  |
| 2,4(2565,00000 (1068,7500 quot. |  |
| 24.. $\cdots$. |  |
| 165 In | In this example, becaufe there |
| 144 | are no decimal figures in the dividend, therefore I annex |
| 210 |  |
| 192 | five cyphers to it, and then |
| 180 | having divided, I find that my quotient is to have four deci- |
| 168 |  |
| 120 | mal places; but the two |
| 12 | noughts at the right may be rejected. |
|  |  |

A Treatije of Guaging.
Divide 231 by ,7854?
,7854) 231,00000000(294,1176 quot. 15708......

| 9240 |
| :--- |
| $\frac{7854}{13860}$ |
| $\frac{7654}{60060}$ |
| $\frac{54978}{50820}$ |
| $\underline{47124}$ |
| $\underline{3696}$ |

Divide, 7854 by 231? 231), 78 8400(3400 $\frac{693}{924}, 003400$ quot. $\frac{144}{160}$ 924
....00
Here, becaufe there are not enough of places

$$
\begin{aligned}
& \frac{144}{160} \\
& \frac{144}{16}
\end{aligned}
$$ in the quotient, therefore I make up the deficiency by prefixing cyphers.

To reduce a Vulgar Fraction to a Decimal.

## Rule.

Annex cyphers to the numerator, and divide by the denominator, and the quotient will be the decimal required.

Examples.


See the work.

$$
\left\{\begin{array}{l}
\frac{\frac{1,00}{, 50} \text { or }, 5}{5)\left(\frac{2,00}{, 40} \text { or }, 4\right.} \\
\frac{3,000}{, 750}=, 75
\end{array}\right.
$$

Reduce $\frac{\frac{25}{3}}{3}$ to a decimal? 32)25,00000(,78125 anfwer.
$\frac{224 \cdots \cdots}{260}$
$\frac{256}{40}$
$\frac{32}{80}$
$\frac{64}{160}$
$\frac{160}{0}$
$\square$

Reduce $\frac{2}{3}$ to a decimal?
3)2,0000
,6666 anfwer.
In this example it appears that there will be a remainder, let the divifion be carried on as far as you pleafe; but four figures. of a decimal being generally exact enough for any purpofe in guaging, the remainder may be dropt as inconfiderable.

Reduce $\frac{396}{558}$ to a decimal? 559) 376,00000(,67262 anfwer.
$3354 . \cdots$

| 4060 |
| :--- |
| 3913 |
| 1470 |
| 1118 |
| 3520 |
| $\frac{3354}{1660}$ |
| 1118 |
| 542 |

Reduce $\frac{\frac{\pi}{\Sigma}}{50}$ to a decimal. 16) $1,00000(6250$ $\frac{96 \ldots, 06250 \text { anfwer. }}{40}$

To reduce inferior denominations to the Fraction of a bigher; as Shillings and Pence, to the Fraction of $a$ Pound; Pints to the Fraction of $a$ Gallon, छัc.

## Rule.

If there be more denominations than one, reduce them to the loweft mentioned, to which annex a fufficient number of cyphers; and divide this by that number which fhews how many of the leffer denomination will make one of the greater; the quotient will be the decimal required. Thus to reduce pence to the decimal of a pound, put cyphers to the number of pence, and divide by 240 , the pence of a pound. In like manner, to bring pints to the decimal of a gallon, annex cyphers to the number of pints, and divide by 8 , the pints of a gallon, and the quotient will be the decimal, $E^{\circ} \mathrm{c}$.

Examples.
Reduce 16 billings to the decimal of a pound? 20) $16,000(, 800$ or , 8 anger.


Reduce 176 to the decimal of a $£_{0}$ ? 17-6
12
240(210,000(,875 answer.


$$
\begin{array}{r}
\frac{720}{800} \\
\frac{720}{80}
\end{array}
$$

Reduce 6 pints to the fraction of a gallon? 8)6,000
,750 answer.
Reduce I pint to the fraction of a gallon?
8) 1,000
,125 anfieer.


To find the value of a decimal; that is, to find bow many of the next inferior denomination, the decimal of a fuperior will make.

Rulc.
Multiply the decimal by that number which fhews how many of the leffer denomination will make one of the greater; and from the product cut off as many figures to the right hand, as there are in the given decimal; the figures on the left of the feparating point will be the number of the faid leffer denomination, and thofe on the right a dicimal thereof, of which find the value as before; and fo on from denomination to denomination till the loweft be arrived at, or till the figures cut off be all cyphers.

Examples.
What is the value of, $6 £$ ? What is the value of, 785 f



A Treatije of Gauging:
What is the value of, 5 of Find the value of, 75 of a gallon?

$$
\frac{95}{4,0} \text { answer } 4 \text { pints. }
$$

Find the value of, 837 of
 N. B., 696 is the fame as $\frac{696}{1000}$ which is a little more than two thirds of a pint.

What is the value of ,3765 of a tun?
 a bog head?
,75
$\begin{array}{r}63 \\ \hline 225\end{array}$
$\frac{450}{47,25}$ Answer $\begin{array}{r}g \cdot p \cdot \\ 47^{-2}\end{array}$

Find the value of ,7687 of a yard?
,7687

$$
\begin{aligned}
& \frac{3}{2,3061} \\
& \frac{12}{3,6732}
\end{aligned} \text { inf. inch. } \quad \begin{gathered}
\text { f. in, } \\
2732
\end{gathered}
$$

What is the value of , 33666 of a gallon? ,33666 8

$$
\underbrace{2,99228}
$$

Answer, 2,99228 pints, which may be called 3 pints, becaufe the decimal is nearly equal to I .

To Square or Cube a Number.
Rule.
Multiply the number by itfelf, and the product is the fquare required ; alfo, multiply the fquare by the number, and this product is the cube.

Examples.
Required the Square of 30 ? Required both Square and

| 30 |
| :---: |
| 30 |
| Square |
| 900 |
| answer. |.

What is the Square and cube of 12,6 ?

12,6
$\frac{12,6}{75^{6}}$
252
126
258,56 Square.
12,6
15525
$5175^{2}$
$\frac{25876}{3260,376}$ cube.
Required the Square of I ? I
$\frac{1}{1}$ Square.

What is the fquare of 2,62 ? | What is the fquare and


Required the Square and cube of, 12 ?


To extract the Square Root.

## Definition.

To extract the fquare root of any given number, is to find a number which being multiplied into itfelf, fhall produce the given number; thus the fqnare root of 25 is 5 , of 49 is $7, \mathcal{E}^{\circ}$.; moreover, 25 is called the fquare of $5, \mathcal{E}^{\circ} c$.

Rule.
Let the following table of fquares and roots be committed to memory.

| Roots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Square | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |

Then divide the given number into periods of two figures each, beginning at the right hand and pointing
pointing to the left, but in decimals reckon from the left hand towards the right, beginning at the decimal point.

Find the neareft lifer root of the left hand perood, place the figure fo found in the quotient, for the firft figure of the root, fubtract its fquare from the faid period, and to the remainder bring down the next period for a dividual or refolvend.

Double the quotient for a divifor; and find how often the divifor is contained in the dividual excluding the right hand figure ; place the figure denoting the anfwer both in the quotient, and for the right hand figure of the divifor, and you have the complete divifor.

Multiply the divifor thus completed by the figure put in the quotient ; fubtract the product from the dividual, and to the remainder bring down the next period for a new dividual, and then proceed as before until every period is brought down; and the quotient is the root required.
$N . B$. If after every period of the given nimber is brought down, there happen to be a remainder, you are to continue the operation, by annexing periods of two cyphers each for a decimal.

Examples.

Required the Square root of 133225 ?
$13^{\prime} 32^{\prime} 25$ ( 365 root. 9 Square.
66)432 dividual.

396 product.
$7 2 5 \longdiv { 3 6 2 5 }$ dividual. 3625 product

| 365 |
| :--- |
| 365 |
| 1825 |
| 2190 |
| 1095 |

13.325 proof.

## Required the Square root of 549,9025 ?

$5^{\prime} 49^{\prime}, 90^{\prime} 25(23,45$ root.
$\frac{4}{43) 149}$
$\frac{3129}{464) 2090}$

$\frac{41856}{4685) 23425}$| 23425 |
| :--- |

$N . B$. The new divifor is eafily found by adding the laft figure as in this example, which method faves the trouble of doubling the quotient.

| Extract the Square root of 356 ? $3^{\prime} 5^{6}$ (18,8679, \&c. root. |  |
| :---: | :---: |
|  |  |
| 28)256 | Here I put double cyphers |
| 224 | to the remainder for a decimal, and carry the |
| 368) 32 |  |
| 2944 | work on until there are four figures of a decimal, which are generally enough for any purpofe in guaging. |
| 3766) 25600 |  |
| 22596 |  |
| 37727) 300400 |  |
| 264089 |  |
| $3 7 7 3 4 9 \longdiv { 3 6 3 1 1 0 0 } \begin{array} { r }  { 3 3 9 6 1 4 1 } \end{array}$ |  |
|  |  |  |
| 234959 | remainder. |

Required

Required the fquare root of, 1296 ? ,12'96(,36 root.
66) $\begin{array}{r}396 \\ 396\end{array}$

Note, The fquare root of a whole number can be extracted whether it have an even or an odd number of figures; but the fquare root of a decimal with an odd number of figures can not be taken, until one, three, $\mathcal{E}^{\circ} c$. cyphers be annexed to it, in order to make each period of it confift of two figures ; as in the following examples:



To extract the Cube Root of a Number.

## Definition.

To extract the cube root of a number, is to find a number which being multiplied by itfelf, and this product multiplied again by itfelf, this laft product fhall be equal to the given number ; thus the cube root of 8 is 2 , and of 64 is $4, \varepsilon^{\circ} c$.
Rule.

Let the following table of cubes and roots be committed to memory:

| Roots | 1 | 213 | 4 | 5 | 6 | \% | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cubes | I) | 1827 | 64 | 125 | 216 | 343 | 512 | 729 |

Then divide the given number into periods of three figures each, beginning at the right hand and pointing to the left ; but in decimals reckon
from
from the left towards the right, beginning at the decimal point.

Find the neareft leffer root of the left hand period, place the figure fo found in the quotient, for the firlt figure of the root; fubtract its cube from the faid period, and to the remainder bring down the next period for a dividual or refolvend. Multiply treble the quotient by the quotient, and call this the defective divifor, and try how often the defective diyifor is contained in the dividual, referving two places on the right of the defective divifor, to be filled up by the fquare of the quotient figure if it have two figures, or by a cyphe: and it, if it have but one; and to this add treble the quotient with a cypher annexed multiplied by the quotient figure, and you have the complete divifor; then multiply the complete divifor by the quotient figure, and fubtract the product from the dividual, and to the remainder bring down the next period for a new dividual, which is to be divided by a divifor found as above; and thus proceed until all the periods are brought down, and the quotient is the root required.
$N$. B. If after ceery period of the given number is brought down, there happen to be a remainder, you are to continue the operation by annexing periods of three cyphers each for a decimal.

The defective divifor may be found by addition, thus; to the laft complete divifor add the number which completed it, together with twice the fquare of the laft quotient figure, and you have the new defective divifor,
Examplcs?

Examples.
Required the cube root of 1281904? $12^{\prime} 812^{\prime} 904$ (234 root.
8
Def. divifor, 1209 ) 4812 dividual.
add, $\frac{180)}{\text { Comp. divif. } 1389) 4167}$
Def. divif. 158716$) 645904$ dividual.
add, 2760$)$

Comp. div. 161476)645904


Here I find the defective divifor according to the firft and fecong methods, where it appears that the latter is by much the eafieft that perhaps can be.

Def. div. $2700 \bar{j}^{6}$ ) 1652616
add, 5400)
Com. div. 275436)16526I6

What is the cube root of $75^{8} 4$ ? $7^{\prime} 5^{84}(\mathrm{I} 9,64,8 \mathrm{cc}$. root. I
II $53^{88} 33^{6)} \frac{46193344}{8270656}$ rem.

Required the cube root of , 36 ? , $360^{\prime} 000^{\prime} 000$ (,7 I I \&c. root. 343
14701) I7000 210)
14911) 14911
1512301) 2089000 2130)
1514431) $\frac{\text { 151443I }}{574 j^{69}} \mathrm{rcm}$.

A few

A few examples for exercife in the fquare and cube roots.

$$
\begin{aligned}
& \left.\left.\begin{array}{l}
\text { Required the } \\
\text { cube root of }
\end{array}\right\} \begin{array}{l}
164566592 \\
387420489 \\
7121,10216
\end{array}\right\} \text { : }\left\{\begin{array}{l}
548 \\
729 \\
19,238
\end{array}\right. \\
& \text { cube root of }\left\{\begin{array}{l}
387420489 \\
7121,10216
\end{array}\right\} \underset{y}{\curvearrowright}\left\{\begin{array}{l}
729 \\
19,2388 \sigma^{\circ} .
\end{array}\right.
\end{aligned}
$$

## 4

С h a f . II.

## Of G E O M E T. R Y.

Definitions.

1.Point is that which hath no parts, or which hath no magnitude.
2. A Line is length without breadth.
3. The Ends or Bounds of a line are points.
4. A Straight Line is that which lieth evenly between its extreme points; or according to fome, a ftraight line is the fhorteft diftance between two points; others fay, a ftraight line is fuch, that if the eye be placed in a continuation of it, then the point or end of the line which is next the eye, will hide the whole line.
5. A fuperficies is that which hath only length and breadth.
6. The Bounds or Extremities of a fuperficies are lines.
7. A Plane Superficies is that which lies evenly between its lines; or it is that in which any two points being taken, the ftraight line between them lies wholly in the fuperficies; or what is the fame thing, a plane fuperficies is fuch, that if the eye be placed in a continuation of it, then the line or extremity of the fuperficies which is next the eye, will hide the whole fuperficies. Note, a plane fuperficies is commonly called a plane; the word superficies being omitted.
8. A Plane Angle is the inclination of two ftraight lines to one another which meet together, but are not in one fraight line; or it is the opening between two ftraight lines: where obferve, that the angle is not increafed by making the lines longer which contain it ; but it is made larger, by opening thefe lines wider.
9. When a ftraight line ftanding on another ftraight line makes the angle on each fide of it equal between themfelves, either of thefe equal angles is a Right one; and that ftraight line which ftands upon the other, is called a Perpendicular to that one on which it ftands.
10. An Obtufe Angle is that which is greater than a right angle.
11. An Acute Angle is that which is lefs than a right angle.
12. A Figure is that which is inclofed by one or more boundaries.
13. A Circle is a plane figure contained by one line called the Circumference, and is fuch that all ftraight lines drawn from a certain point within the
the figure to the circumference are equal between themfelves.
14. And this point is called the Centre of the Circle.
15. The Diameter of a Circle is a ftraight line, drawn through the centre, and terminated both ways by the circumference. Note, Half the diameter, to wit, the ftraight line drawn from the center to the circumference, is ufually called the Radius of the Circle.
16. A Semcircle is the figure contained by a diameter and the part of the circumference cut off by the diameter.
17. A Segment of a Circle is the figure contained by a ftraight line, and the part of the circumference which that ftraight line cuts off. Or the definition may be thus; when a ftraight line paffes through a circle, it divides the fame into two fegments. Hence, if the line paffes through the center, then each fegment is a femicircle, for the line in that cafe is a diameter; but if the line does not pafs through the center, one of the fegments is greater than a femicircle, and the other one is lefs.
18. Straight-lined Figures are thofe which are contained by ftraight lines.
19. Three-fided Figures or Triangles by three ftraight lines; Four-fided by four; E®c. but figures which are contained by more than four ftraight lines, are ufually called Polygons or many-fided figures.
20. Of three fided figures, that is an Equilate-
ral Triangle which hath its three fides equal ; that an Ifofceles Triangle which hath two equal fides; and that a Scalene Triangle which hath its three fides unequal.
21. A Right-angled Triangle is that which hath one right angle and two acute ones; an Obtufeangled Triangle hath one obtufe angle and two acute ones; and an Acute-angled Triangle hath all its angles acute.
22. Of four-fided figures, that is a Square which hath all its fides equal, and all its angles right ones; that an Oblong or Rectangle which hath all its angles right ones, but its oppofite fides only equal; that a Rhombus which hath all its fides equal, but its angles are not right ones; and that a Rhomboid which hath its oppofite fides equal, but all its fides are not equal, nor its angles right ones. Note, Thefe four different figures, to ruit, the fquare, oblong, rhumbus, and rhomboid, are in general called Parallelograms. All other four fided figures befides thefe, are called Trapeziums.
23. Parallel ftraight lines are fuch as are in the fame plane, and which being produced ever fo far both ways do not meet ; and Parallel Planes are fuch as being produced infinitely on all fides do not meet.
24. A Solid is that which hath length, breadth and thicknefs.
25. That which bounds a folid is a Superficies.
26. A Straight Line is perpendicular, or at right angles, to a plane, when it makes right an-
gles with every fraight line meeting it in that plane.
27. A Solid Angle is that which is made by the meeting of more than two plane angles in one point, provided thefe plane angles are each in feparate planes.
28. Similar rimht-lined tigures are thofe which have their feveral angles equal, each to each, and the fides about the equal angles proportionals.
29. Similar Solids are fuch as have all their fod lid angles equal, each to each, and which are contained by the fame number of fimilar planes.
30. A Cube is a folid contained by fix equal fquares ; that is, it hath its length, breadth, and depth equa! ; or to be familiar, dice are cubes.
31. A Paraiielopipedon is a folid contained under fix paral elograms, whereof every oppofite two are parallel ; when the fix figures which contain the folid are right anged, whether they be fquares or oblongs, then it is faid to be a right angled parallelopipedon; or to be familiar, chefts; boxes, joices, fuuared beams, $E \sigma^{\circ}$. are parallelopipedons.
32. A Pyramid is a folid contained by feveral triangular planes fet upon one plane, and meeting together in a point; that is, a tapering folid which ends in a point; or a pointed wedge. Note, The plane upon which the triangles are fet, may be any ftraight-lined figure, as a triangle, fquare, oblong, $\varepsilon^{\circ} c$.
33. A Cylinder is a folid defcribed by the motion of an oblong or rectangle about one of its fides;
fides which remains at reft; and that fixed line about which the rectangle turned, is called the Axis of the Cylinder. Rolling ftones, ufed by gardeners, drums, round rulers, $\xi^{\circ} c$. are cylinders. 34. A Cone is a folid defcribed by the revolution of a right-angled triangle, about one of the fides which contain the right angle, the faid fide remaining at reft; and this fixed fide, about which the triangle moved, is called the Axis of the Cone; that is, a cone is a round tapering folid which ends in a point. Thus, a fugar loaf is a cone.
35. The Fruftum of a Cone, is that part which remains, the top being cut away; thus all yeffels which have their fides ftraight, and are wider at one end than at the other, are fruftums of a cone. After the fame manner, if the top part of a pyramid be cut off, the remainder is a fruftum thereof.
36. A fphere or globe is a folid defcribed by the revolution of a femicircle about the diameter which remains at reft; and the diameter which remains fixed is called the Axis of the Sphere. Balls and all other round folids are fpheres.
37. An Eilipfis or Oval is a plane figure bounded by a regular curve line, returning into itfelf; and it differs from a circle in this, that its diameters are not equal to each other ; of its longeft and fhorteft diameters which cut other at right angles in the center, the former is called, the Tranfverfe diameter, the other the Conjugate.

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An ellipfis is made by the oblique fection of a cone, or of a cylinder.
38. A Spheroid is a folid defrribed by the motion of a femi-ellipfis about one of the diameters remaining at reft; and this fixed line about which the figure revolved, is called the Axis of the Spheroid; moreover, if the axis be the tranfverfe or longef diameter, then it is called a Prolate Spheroid; but if it be the conjugate, then it is an Oblate Spheroid. A prolate fpheriod refembles an egg, but an oblate a turnip.
39. The Middle Zone of a fphere, or fpheroid, is that part which remains, the two ends of the folid being cut off; and thefe ends or parts cut off are called fegments; thus a cafk is the middle zone of a prolate fpheroid; moreover, bowls are fegments of fpheres, or fpheroids.
A few Principles belonging to Geometry, Menfuration, and Gauging, which the Learner תhould make binjfelf acquainted with.

1. Every magnitude is meafured by fome magnitude of the fame kind. A line, by a lineal foot, yard, $E^{\circ} c$. A fuperficies, by a fquare foot, yard, $\mathcal{E}^{\circ} c$. A folid, by a cubic foot, yard, $\sigma^{\circ} c$. Note, Magnitude is a general erm, forlines, fuperficies, or folids; and hence there are faid to be three kinds of magnitudes.

Lineal or running meafure is known to all, and needs neither direction nor example. There remain then only the fuperficial and folid meafure to be explained.
2. The number of fquares which any fuperfi-
cial figure contains, is called its Area or Content; and the nurnber of cubes which any folid figure contains, is called its Solidity or Content. Hence when the area of a figure is demanded, the thing to be done is to find the number of fquares which that figure contains; and in like manner, when the folidity of any folid is required, the thing to be done is to find the number of cubes which the propofed folid figure contains; and in order to find the area of any fuperficies or folidity of a folid, the proper dimenfions muft always be given.
3. The area of a fquare is found by multiplying the fide into itfelf, that is, by fquaring the fide: Thus, if the fide of a fquare be eight feet, then its area or content is $6_{4}$ fquare feet.
4. The area of an oblong or rectangle is found by multiplying the length by the breadth: Thus, if the length of an oblong be 9 yards, and its breadth 6 , then its content is 54 fquare yards.
5. Every right-angled triangle is equal to half a rectangle, whofe length is one of the fides of the triangle which contain the right angle, and the breadth the other fide. Wherefore, if thefe two fides, (ufually called the Bafe and Perpendicular) be multiplied together, the product will be double the content of the triangle: Thus, if the bafe be 10 feet, and the perpendicular 8, then the area of the triangle is 40 fquare feet, to wit, the half of 10 times 8 .
6. In any triangle, whether it be right, obtufe, or acute angled, if you multiply the bafe by half the perpendicular, or the perpendicular by half
the bafe, or laftly the whole bafe by the whole perpendicular, and take half the product ; any of thefe methods will give the area of the triangle Note, Any fide of a triangle may be cailed the bafe, and the perpendicular is to fall from the angle oppofite to the bafe, either upon the bafe, or upon it produced, if it does not fall within the triangle.
7. The area of a rhombus, or rhomboid, to wit, any inclining parallelogram, is found by multiplylng the length by the perpendicular breadth.
8. The area of a trapezium, or any four fided figure whatfoever, is found by multiplying the diagonal by half the fum of the two perpendiculars, or the fum of the perpendiculars by half the diagonal, or the diagonal by the whole fum of the perpendiculars, and taking half the product; any of thefe methods gives the content. Note, The diagonal of a trapezium is a ftraight line drawn from any one of the angles of the figure to its oppofite one; and the two perpendiculars are to fall from the other two angles upon the diagonal, or upon it produced, if need be.
9. The area of a many fided figure is found by dividing it into trapeziums and triangles, and and finding the areas of thefe trapeziums and triangles by the foregoing articles, then the fum: of thefe will be the content of the figure; that is, add together the contents of the trapeziums or triangles which make up the given polygon, and you have its area required.

10 . Tbe diameters of circles are proportional
to their circumferences ; that is, as the diameter of any circle is to its circumference, fo is the diameter of any other circle to its circumference ; now it is known, that,
If the diameter $\left.\left.\left\{\begin{array}{r}7 \\ 113 \\ i\end{array}\right\} \begin{array}{c}\text { its cir- } \\ \text { cumfer- }\end{array}\right\} \begin{array}{l}22 \\ \text { ence is }\end{array}\right\} \begin{aligned} & 355 \text { more nearly } \\ & 3,1416 \text { fill more }\end{aligned}$ (nearly:
Hence, as 7 is to 22 , or as 113 is to 355 , or as 1 is to 3,1416 ; fo is the diameter of any circle to its circumference; and inverfely, as 22 is to 7 , or as 355 is to 113 , or 3,1416 is to 1 ; fo is the circumference of any circle to its diameter. Wherefore if either the diameter or circumference of a circle be given, the other can be found by the rule of three.
11. The area of a circle is found by multiplying. half the diameter by half the circumference; or if the diameter only be given, then the area may be found independently of the circumference ; thus, multiply the fquare of the diameter by, 7854 , and the product is the area; or, take $\frac{\frac{x}{2}}{2}, \frac{x}{4}$, and $\frac{5}{T}$, of this fourth of the fquare of the diameter, and you have the area. Note, , 7854 is $\frac{5}{4}$ of 3,1416 , or it is the area of a circle whofe diameter is 1 ; or if the circumference be given, then the area may be found by firft finding the diameter, and then multiplying half the diameter by half the circumference; or independently of the diameter, by multiplying the fquare of the circumference by, 07958 . Note, , 07958 is the area of a circle whofe circumference is :, or it is, 25 divided by 3,1416 .
12. If the area of a circle be given to find the diameter, or circumference; multiply the area of the circle by $1,273^{2}$, or divide it by, 7854 , and the fquare root of the product or quotient will be the diameter; alfo multiply the area of the circle by 12,5664 , or divide it by, $0795^{8}$, and the fquare root of the product or quotient will be the circumference.
13. The fquare root of the area of any figure, will be the fide of a fquare equal in content to the given figure. Hence to find the fide of a fquare which fhall be equal to a circle or 10 any other figure, we have only to find the area of the propofed figure by the foregoing articles, then the fquare root of this being extracted, will be the fide required. If the area of a rectangle be divided by one of the fides, the quotient will be the other.
14. The area of an ellipfis is found by multiplying the tranfverfe and conjugate diameters together, and that product by, 7854 .
15. The folidity of a cube is found by multiplying the fide by itfelf, and this product again by the fide, that is, by cubing the fide; or what is ftill the fame thing, by multiplying the length, breadth, and depth together; thus, if the fide of a cube be 4 feet, its folidity is 64 cubic feet.
16. The folidity of a right-angled paralielopipedon is found by multiplying the length, breadth, and depth together ; thus, if the length be 8 , the breadth 6 , and the depth 4 feet; then the folidity is, 192 cubic feet.
17. The folidity of a cylinder is found by firft finding
finding the area of the circular bafe, and multiplying this by the length or height, the product will be the folidity required ; that is, multiply the fquare of the diameter of the bafe, by, 7854 , and this product by the length, and you have the folidity.
18. The folidity of a pyramid, or of a cone, is found by multiplying the area of the bafe by $\frac{\frac{1}{3}}{}$ part of the perpendicular altitude; that is, in a cone by multiplying the fquare of the diameter of the bafe by, 7854 , and this by $\frac{\pi}{3}$ part of the axis, or the perpendicular let fal from the vertix to the plane of the bafe. For every pyramid is $\frac{\pi}{3}$ part of a prifm which has the fame bafe and altitude, or length, with the pyramid ; and every cone is $\frac{\frac{\pi}{3}}{3}$ part of its circumfcribing cylinder. N. B, A Prifm is a folid which does not taper; it is contained by planes whereof two are equal and parallel, called its Bafes, and the other planes are parailelograms; hence every parallelopipedon is a prifm ; but every prifm is not a parallelopipedon; becaufe the bafes of a prifm may be triangles, quadrangles, $\sigma^{\circ} c$. Now it is manifeft from the 16 th and 17 th articles, that the folidity of a prifin is found by multiplying the area of the bafe by the length, or perpendicular bet ween the planes of its bafes; and hence $\frac{\pi}{3}$ part of this is the folidity of the pyramid.
19. The folidity of the frufum of a pyramid if it have fquare bafes, is found by miltiplying the two fides of the bafes together, and to this adding ${ }^{\frac{\pi}{3}}$ part of the fquare of their difference, and multiplying this fum by the height; but if the bafes
be any other figures than fquares; multiply the areas of the two bafes together, and to the fquare root of the product add thefe two areas, multiply this fum by $\frac{1}{3}$ part of the fruftum's height, and the product is the folidity.
20. The folidity of the fruftum of a cone is found by multiplying the greater and leffer diameters together, and adding to this $\frac{7}{3}$ part of the fquare of their difference, and multiplying this fum by the highit or axis, and this product being multiplied by, 7854 will give the folidity.
21. The folidity of a globe is equal to $\frac{2}{5}$ of its circumfribing cylinder; hence its folidity is found by multiplying the cube of its diameter by $\frac{2}{3}$ of 27854 , that is, by, 5236 .
22. The folidity of a fpheroid is equal to $\frac{2}{3}$ of its circumfrribing cylinder; hence its content is found by multiplying the diameter about which the femiellipfis revolved, by the fquare of the other diameter, and this by, 5236 , the laft product will be the content.
23. To find the folidity of the fruftum or polar fegment of a globe; add together the fquare of the bafe diameter, the fquare of the height, and $\frac{1}{3}$ part the fquare of the heighth; then multiply this fum by half the height, and that product again by ,7854, this lait product will be the folidity.
24. The folidity of the middle fruffrum or middlle zone of a fphere, or of a fpheriod, is found by adding the fquare of the bafe or end dianueter, to twice the fquare of the diameter of the greateft circle, (to wit, that one in the middle
die of the zone, ) and multiplying this fum by the length or height, and that product by $\frac{\frac{\pi}{3}}{}$ part of ,7854, viz. ,2618; this laft product will be the folidity.
25. The cube root of the folidity of any folid figure, will be the fide of a cube equal in content to the given folid. Hence to find the fide of a cube which fhall be equal to a fphere, cylinder, or any other given folid whatfoever, we have only to find the folidity of the propofed folid by the foregoing articles, then the cube root of this being extracted, will be the fide of the cube required.
26. All fimilar plane figures are to one another as the fquares of their correfpondent lines; that is, as the area of any figure, is to the fquare of a line belonging to that figure; fo is the area of a figure fimilar to the former, to the fquare of a line in this figure correfpondent to that taken in the other. And inverfely, as the fquare of a line in any figure, is to the area of that figure; fo is the fquare of a correfpondent line in a fimilar figure, to the area of that fimilar figure. Hence circles are to one another as the fquares of their diameters, $\varepsilon^{2} c$.
27. All fimilar folid figures are to one another as the cubes of their correfpondent lines; that is, as the folidity of any folid, is to the cube of a line belonging to that folid; fo is the folidity of a folid fimilar to the former, to the cube of a line in this folid correfponding to that taken in the other. And inverfely, $\sigma^{\circ} c$, as in the foregoing article.
article. Hence the folidities of globes are to one another as the cubes of their diameters, $\mathcal{E}^{\circ} c$.
28. If the folidity of any propofed folid be found (by the help of the foregoing articles) in cubic inches, and this be divided by the number of cubic inches in a gallon; then the quotient will be the content in gallons of that folid: And to make this matter eafy and familiar, will be the fubject of the following chapter.

## 

С н а р. III.

## PARTICULARLY of GAUGING.

## Definitions.

1. AUGING is that art which teacheth how to find the content of any veffel in gallons, bufhels, $\xi^{\circ}$ c.; from having the proper dimenfions of that veffel given.
2. The dimenfions are always taken in inches and tenths of an inch.
3. An ale gallon contains 282


Problenm

## Problem 1.

To find the content in ale, wine gallons, $\mathcal{E}^{\circ} c_{0}$ of a fquare ciftern, cooler, cheft, box, EFc. viz. any folid in the form of a right angled parallelopipedon.

> Rule:

Multiply the length, breadth, and depth together ; and divide the laft product by the number of cubic inches in a gallon, or bufhel, and the quotient will be the content required.

## Examples.

1. Required the content in ale gallons of a cif= tern, or cooler for wort, in the form of a rightangled parallelopipedon, whofe length is 200 inches, breadth 150 , and depth 10 ?

200
-150
10000
200
30000
10
282) $300000(1063,8$ content.

| $\frac{282 \cdots}{1800}$ |
| ---: |
| $\frac{1692}{1080}$ |
| $\frac{846}{234,0}$ |
| $\frac{256}{84}$ |

Here the content is 1063 gallons and , 8 of another gallon, which is fomething more than 6 pints.
2. What is the content in wine gallons of a fquare trough, or a right angled parallelopipedon, whofe length is 37,5 breadth 20,2 and depth 16,5 inches.


Problem 2.
To find the content of a veffel, trough, ciftern, $\xi^{\circ} c$. in the form of the fruftum of a fquare pyramid ; that is, a fquare folid which is wider at the one end than at the other.

## Rule.

Multiply the fides of the two bafes together, and to this product add $\div$ part of the fquare of their difference ; then multiply this fum by the depth, and divide by the cubic inches in a gallon, © $^{\circ}$.; the quotient will be the content.

Examples.

## Examples.

1. Required the content in wine gallons, of a veffel in the form of the fruftum of a fquare pyramid, having each fide of the greater bafe 136,3 , each fide of the leffer bafe 104,8, and the depth 75,2 inches?

$$
\left.\begin{array}{rr}
136,3 & 136,3 \\
104,8
\end{array}\right)
$$

$2 3 1 \longdiv { 1 0 9 9 0 4 7 , 2 4 8 ( 4 7 5 7 , 7 \text { content. } }$
2. Required the content in Englifh bufhels of a malt fteep, being the fruftum of a fquare pyramid, having the fide of the greater bafe at the mouth 150 , the fide of the leffer bafe at the bottom 120, and the depth 60 inches?

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| $15^{\circ}$ | 150 |
| :---: | :---: |
| 120 | 120 |
| 3000 | 30 |
| 150 | 30 |
| 18000 | 3)900 |
| 300 | 300 |
| $\begin{array}{r} 18300 \\ 60 \end{array}$ |  |

$2150,4) 1098000,000(510,60$ content.

$$
\begin{array}{r}
107520 \cdots \\
\hline 22800 \\
21504 \\
129600 \\
12902 \frac{4}{5} \\
\hline 5760
\end{array}
$$

To find the content of the fruftum of a pyramid which hath any kind of a figure for its bafe; you muft find the area of each bafe of the fruftum, and to the fum of thefe two areas add the fquare root of their product; then multiply this by $\frac{\pi}{3}$ part of the depth, and divide by the number of cubic inches in a gallon, $\sigma^{\circ} c$, and the quotient will be the content. Moreover the content of a prifm, that is, a folid of equal thicknefs, is found, by firft finding the area of either bafe, and multiplying this by the length, and diriding by the number of cubic inches in a gallon, $\sigma^{\circ} c$.

How

How to find the area of any ftraight-lined figure, has been flown in the lat chapter.

## Problem 3.

To find the content in ale, wine gallons, $\delta^{\circ} c$. of a cylinder.

$$
\text { Rule } \mathrm{I} .
$$

Multiply the fquare of the diameter of the bafe by , 7854 , and this product by the length; then divide by the number of cubic inches in a gallon, $\varepsilon^{\circ} c$. and the quotient is the content.

Or, what is much better, by

## Rule 2.

Multiply the fquare of the diameter of the bale by the length; then multiply or divide by the proper multiplier or divifor ; and the product or quotient will be the content required. Note, The multipliers are found by dividing ,7854 by the number of cubic inches in a gallon or bushel; and the divifors are found by dividing the cubic inches in a gallon, $E^{\circ} c$. by, 7854 .

See the following work for Multipliers. 282), 785400 (,002785 Multiplier for ale gallons. $231), 785400(, 003400$ $268,8), 785400(, 002922$ $2150,4), 785400(, 000365$ wine gallons. gallons dry meafure. Englifh bufhels. Work for Divisors.
,7854)282,00000(359,0 Divifor for ale gallons.
,7854)231,00000(294,12 ,7854)268,80000(342,2
$, 7854) 2150,4000(27,38,0$
gallons dry meafure. Englifh bufhels. Examples.

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## Examples.

1. Required the content in wine gallons of a cylinder whofe bafe diameter is 56,5 , and its length 96 inches?

|  | By Rule 1. |
| :---: | :---: |
| 56,5 | 2507,19315 |
| 56,5 | 96 |
| 2825 | 1504315890 |
| 3390 | 2256473835 |
| $2825 \quad 231$ | ) 240690,54240 ( $\mathrm{ro4r}, 9504 \mathrm{cont}$. |
| 3192,25 | $231 \times \cdots \cdots$ |
| ,7854 | 969 |
| 1276900 | 924 |
| 1596125 | 450 |
| 2553800 | 231 |
| 2234575 | 2195 |
| 2507,193150 | 2079 |
|  | 1164 |
|  | 1155 |
|  | 924 |
|  | 924 |
|  | $\bigcirc$ |

By Rule 2.

Method ift, by the Multiplier.

Method 2d, by the Divifor.


From the foregoing work it appears, that the method according to Rule 2, by multipliers, is by much the fhorteft, efpecially in wine gallons, where the multiplier , 0034 is fuch an eafy number, that the operation will not have more than half the figures which other methods have. Wherefore all the following queftions for finding the contents of cylinders, fruftums of cones, or any other round folid fhall be performed by multipliers.
2. Required the content in ale and wine gallons of a cylinder, whofe bafe diameter is 40 , and length 50 inches?

| 40 |
| :---: |
| $\frac{40}{1600}$ |
| $\frac{50}{80000}$ multiplier. |
| , 0034 |
| 220000 |
| 24,0000 |
| 272,0000 |
| cont. in w.g. |


| $\frac{80000}{, 002785}$ multiplier. |
| :---: |
| 400000 <br> 640000 <br> 560000 <br> 160000 |
| 222,80000 cont. in a.g. |

3. Required the content in Englifh bufhels of a cylinder, whofe diameter is 62,2 , and length 71,3 inches?


## Problem 4.

To find the content of a veffel in the form of the fruftum of a cone; that is, a fraight ftaved veffel, wider at the one end than at the other.

## Rule.

To the product of the two diameters at the top and bottom, add $\frac{\frac{\pi}{3}}{3}$ part of the fquare of their difference; then multiply this fum by the length, and this product being multiplied by the proper multiplier, or divided by the divifor, will give the content required.

## Examples.

I. Required the content in wine, and ale gallons, of a veffel in the form of the fruitum of a cone, whofe bottom diameter is 80 , top diameter 50 , and length 70 inches?

| 80 | 80 | 30100 |
| :---: | :---: | :---: |
| 50 | 50 | ,00278 |
| 4000 | 30 | 150500 |
| 300 | 30 | 2408000 |
| 4300 | 3) $\longdiv { 9 0 0 }$ | 2107000 |
| 70 |  | 602000 |
| 301000 |  | 838,285000 |
| $\frac{, 0034}{1204000}$ | fier. | 830,205000 |
| 903000 |  |  |

2. Required the content in wine gallons of a seffel in the form of a fruftum of a cone, having the diameter of its greater bafe 20,5 , that of the leffer 15,2 , and its length 30,1 inches?

| 20,5 | 20,5 | 9660,896 |
| :---: | :---: | :---: |
| 15,2 | 15,2 | ,0 034 |
| 410 | 5,3 | 38643584 |
| 1025 | 5,3 | $\begin{gathered} 30043504 \\ 28982688 \end{gathered}$ |
| 205 | 159 |  |
| 311,60 | 265 | 32,8470464in w.g. |
| 9,36 | $3 \longdiv { 2 8 , 0 9 }$ |  |
| 320,96 | 9,36 |  |
| $3 \mathrm{O}, 1$ |  | In this example the |
| 32096 |  | content is 32 gal |
| 962880 |  | lons, and 7 pints |
| 9660,896 |  | nearly. |

Problem 5.
To find the content of a vefiel in the form of the fruftum of an elliptical cone; that is, a ftraight faved veffel, wider at the one end than at the other, and having oval bafes.
Rule.

Multiply the tranverfe diameter of any one of the bafes by the conjugate of the other, to this add the two products of each tranfverfe diameter by its own conjugate; then multiply the fum of thefe three products by $\frac{1}{3}$ part of the depth, and this product being multiplied or divided, by the proper
proper multiplier or divifor, the product, or quotient, will be the content. Note, The multipliers and divifors in this cafe are the fame as before, that is, they are thofe which belong to the cylinder.

## Examples.

1. Required the content in wine gallons, of a veffel in the form of the fruftum of an elliptical cone, having the tranfverfe and conjugate diameters of its greater bafe 60 and 45 , and thofe of the leffer $4^{8}$ and 36 , and the depth 72 inches?

$$
\begin{aligned}
& \begin{array}{r|r|r|r}
45 & 48 \\
60 & \frac{36}{288} & \begin{array}{c}
36 \\
2700
\end{array} & \frac{60}{2160} \\
& \frac{144}{1728} & \frac{1728}{24} & \\
& & \frac{1700}{6588} \text { fum of the } 3 \text { prod. }
\end{array} \\
& \overline{2635}^{24}{ }^{\frac{1}{3}} \text { part of depth. } \\
& \frac{13176}{15^{8} 112} \\
& \frac{, 0034}{632448}{ }^{\text {multiplier. }} \\
& \frac{474436}{537,5808} \text { content. }
\end{aligned}
$$

2. Required the content in wine gallons of an oval tub, being the fruftum of an elliptical cone, the tranfverfe and conjugate diameters of its greater bafe are 20,3 and 17,4 , and thofe of the leffer 16,8 and 14,2 , and the depth 25,6 inches?
$3) 25,60$
8,53 $\left|\begin{array}{c}20,3 \\ \frac{17,4}{812} \\ 1421 \\ 203 \\ 353,22\end{array}\right|$

$$
\begin{aligned}
& \frac{292,32}{884,10} \\
& \frac{8,53}{265^{2} 3} \\
& 44205 \\
& 70728 \\
& \text { 7541,373 } \\
& \begin{array}{r}
5003 \\
\hline 305492
\end{array} \\
& \frac{22624019}{25,6405682} \text { content. } \\
& \text { Problemi } 6 .
\end{aligned}
$$

To find the content of the fruftum, or polar fegment, of a fphere.
Rule.

To the fquare of the diameter of the bafe, add the fquare of the depth, together with $\frac{\frac{1}{3}}{}$ part of the
the fquare of the depth, then multiply the fum of thefe three by half the depth, and this product being multiplied by the proper multiplier, or divided by the divifor, the product or quotient will be the content.

## Examples.

1. Required the content in wine gallons, of a bowl in the form of the polar fegment of a globe; the diameter of the mouth of the bowl is 18 , and the depth 12 inches?

$$
\begin{aligned}
& \begin{array}{r|r|r}
18 \\
18 \\
\hline 144 & 12 & 2) 12 \\
\frac{18}{324} & \frac{12}{144} &
\end{array} \\
& \frac{48}{516} \\
& \frac{6}{3096} \\
& \frac{, 0034}{12384} \text { multiplier. } \\
& 9288 \\
& 10,5264 \text { contents }
\end{aligned}
$$

2. Required the content in ale gallons of $a$ punch bowl, in the form of a fegment of a fphere; whofe diameter at the mouth is 15,3 and depth 9,5 inches?

| 15,3 | 9,5 | 2) 9,50 | 354,42 |
| :---: | :---: | :---: | :---: |
| ${ }^{15,3}$ | 9,5 | 4,75 | 4,75 |
| 459 | 475 |  | 177210 |
| 765 | 855 |  | 248094 |
| $\pm 53$ | 3) $\longdiv { 9 0 , 2 5 }$ |  | 141768 |
| 1534,09 90,25 | 30,08 |  | $\begin{gathered} 1683,4950 \\ , 00278 \end{gathered}$ |
| 30,08 |  |  | 13467960 |
| 354,42 |  |  | $\begin{aligned} & 11784465 \\ & 3366990 \end{aligned}$ |
|  |  |  | 4,68011610 |

## Problem 7.

To find the content of an irregular bowl, or of the bottom part of a furnace, ftill, $E^{\circ} c$.

## Rule.

Fill the bowl, or bottom of the furnace, $E_{0} c_{0}$ with water, and meafure the diameter of the furface of the water in feveral places, and in like manner meafure the depth of the water in feveral places; then add the feveral diameters meafured together, and divide the fum by the number of diameters, and you will have the mean diameter of the furface ; alfo add all the depths together, and divide the fum by the number of depths more one, and the quotient will be the mean depth; then with the mean diameter, and mean depth, find
find the content as if it were a cylinder, and the thing propofed will be done.

## Example.

The following dimenfions were taken, in order to find the content in wine gallons, of the bottom part of a large ftill, which was previoufly filled with water, viz.
Diameters $\left\{\begin{array}{l}54,5 \\ 54,9 \\ 54,7 \\ 5)\end{array}\right.$ Depths $\left\{\begin{array}{r}8,3 \\ 16,7 \\ 12,4 \\ 8,4 \\ 4,2\end{array}\right] \begin{array}{r}5,60,0\end{array}$
I defire to know the content?
The 3 diameters being added together and divided by 3 , gives 54,7 for the mean diameter ; and the 5 depths being added together and divided by $\epsilon$, gives 10,0 for the mean depth; hence the content is ror gallons and 6 pints.

Note, If the water have any confiderable depth clofe by the edge of its furface; then this depth being taken together with the feveral others; the fum of thefe is to be divided by the number of depths, in order to have the mean depth, and not by their number more one, for this is only to be done in cafes where the water has no depth at the fide.

Problema

## Problem 8.

To find the content of a cafk, having the bung and head diameters, and the length given.

## Rule.

Multiply the difference between the bung and head diameters, by the number which belongs to that cark in the following table; add the product to the head diameter, fo will the fum be the diameter of a cylinder of the fame length with the cafl, and having the fame content; hence if the content of this cylinder be found, it will be the content of the cafk.

## A Table of Multipliers, for

Rum puncheons, and all $\}$
calks fimilar to them. $\}, 67$
Wiue pipes, and all fimi-
iar cafis.
Cafls having the diffe-
rence between their
head and bung diame- $\}, 6$ ters, leifs than $\frac{1}{\text { ro }}$ part
of the head diameter.
That is, mutiply the difference between the bung and head diameters by thefe multipliers, and add the product to the head diameter and you have the mean diameter; then multiply the fquare of the mean diameter by the length, and this product by ,0034 for wine gallons, $\sigma^{\circ} c$. and you have the content.
The above table is the refult of meafuring with an exact gallon feveral times over, a great num.

A Treatife of Gauging.
ber of cafks, and taking a mean among all thofe of the fame form; and it may be prefumed, that if the dimenfions of any cafk be taken with care, this method will give its content much truer than it could be meafured by a gallon, as it will never be wrong one gallon in 200, but in many inflances it will come within one pint of the truth in a rum puncheon of 130 gallons content.

## Examples.

1. Required the content in wine gallons of a rum puncheon whofe head diameter is $2 \%, 6$, bung diameter 33,6 , and length 38 inches.

2. Required the content in wine gallons of a wine pipe, whole bung diameter is 33,8 , head dimeter 25,6 , and length 52,3 inches ?

| 33,8 |
| ---: |
| 25,6 |
| 8,2 |
| , 64 |
| 328 |
| 4,92 |
| 5,248 |

$$
\begin{aligned}
& \text { 25,6 } \\
& \frac{5,25}{30,85} \text { mean diam. }
\end{aligned}
$$

$$
\begin{aligned}
& 28551675 \\
& { }^{1} 903445^{\circ} \\
& \frac{47586125}{49775,08675}
\end{aligned}
$$

In this example it may be obferved, that after I multiply by, 64 , I call the product 5,25 , inftead of $5,25^{8}$, by which I Shorten the work, and at the fame time have the content within the $\frac{10}{\text { roo part of a pint of what it would have }}$ been, had I not made this alteration; moreover, I have rejected all the figures of the fraction, (for the fame reafon), before I multiply by the multiplier , 0034 .
3. What is the content of a cafk in wine, and ale gallons, whofe bung diameter is 27,2 , head diameter 25,1 , and length 30 inches?
27,2
$\frac{25,1}{2,1}$
$\frac{, 6}{1,26}$

$$
\begin{aligned}
& \text { 25, } 1 \\
& \frac{1,26}{26,36} \text { mean diam. } \\
& \begin{array}{l}
\frac{26,36}{15816} \\
7908
\end{array} \\
& 15816 \\
& \text { 70,87470 in wine gall. } \\
& \text { 20845,5 } \\
& \frac{30}{20845,4880} \left\lvert\, \frac{, 002785}{58,0547175 \text { in ale gall. }}\right.
\end{aligned}
$$

s To a careful reader the foregoing rules and examples will be found to be fully fufficient to make him acquainted with all the neceffary parts of gauging, as performed by calculation ; and in regard to the methods of finding the contents of veffels by the flide rule or diagonal gauging rod, it is full time to fet them afide, as a fufficiency of mifchief has already been done by thefe erroneous inftruments.
N. B. Ullare gauging, that is, finding the content of a calk wobichb bas a part of the liquor drawn out of it, flall be foown whon we come to explain the ufe and con/truction of the tables.

A TABLE for finding the Mean Diameters of Rum

 , 3 16,5 5 I $6,6|16,6| 16,6|16,7| 16,7|16,7| 16,8|16,8| 16,8$
 , $6116,7|16,8| 16,8,16,8|16,9| 16,9 \mid 16,9.17,0117,0117,0$





 , $6,7117,417,517,517,51{ }^{1} 7,617,617,617,7,17,711,7$
,8117,517,6117,617,6117,717,717,717,817,817,8

 ,2,317,817,917,9:17,9 18,018,018,018,118,1:18,1






, 3 18, $6_{1}$ 18, $6118,6|18,7| 18,7|18,7,18,7| 18,8,18,8 \mid 18,8$
 , 618,8 18,8 18,8 18,9 $18,918,9$ 19,0 19,0 19,0 19,0





,5 10,4 19,4 49,4 i9,5 $19,5: 19,5$ 19,6 19,6 19, 619,7



 $, 2,319,919,919,920,0: 20,0|20,020,1| 20,1|20,1| 20,2$


,7120,0 20,2 20, 220,3 20, $3^{20}, 3|20,4| 20,4 \mid 20,420,5$

Puncheons or Cafks of the Firft Form.



,9 , 918,0 18,0
19,0,1 18,0 , 1 , 18
, 2

,9,20,
$\begin{array}{rrr}20 & , 1 & , 7 \\ , 2 & , 3 & , 8 \\ , 8 & ,\end{array}$
, 4 '919'0 19 '

$21^{\circ}$
10
3

$, 7,\left.8\right|_{20,9}, 9,921,020,020$,
22,0,'
, 3
$, 6,7$
, 8
,9,23
${ }^{2} 3, \quad, 1$

| $, 5,6$ | , 1 | , 1 | , 1 | , 2 | , 2 | , 2 | , 3 | , 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| , 3 | , 3 | , 4 |  |  |  |  |  |  |
| , 2 | , 2 | , 2 | , 2 | , 3 | 2 | , 3 | , 4 | , 4 |
| , 4 | , 5 |  |  |  |  |  |  |  |

## AT A B L E for finding the Mean Diameters of Rum



| ,1,2 | ,4 | , 5 | , 5 | , 5 | , 6 | , 6 | , 6 | , 7 |  | , 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ,3 | , 5 | ,6 | ,6 | , 6 | , 7 | , 71 | , 71 | , 8 | , 8 | , 8 |
|  | , 6 | , 7 | , 7 | , 7 | , 8 | , 8 | , 8 | ,9 | , ${ }^{1}$ | -9 |
| , 6 | , 7 | ,8 | ,8 | , 8 | ,9 | ,9 |  | :9,0 |  | 19 |






Puncheons or Cafks of the Firf Form.
 , 1 ,
.3
4
, 7
23,0
$, 9 \quad, 922,022,022,0 \quad, 1 \quad, 1 \quad, 1$

$$
, 3
$$

,6
$\begin{array}{rrr}, 6 & , 7 & , 4 \\ , 8 & , 5 \\ , 94 & , 0\end{array}$

| 22,0 |  |
| ---: | ---: |
| , 2 | , 1 |

$, 5,3$
$24,1,7,8,8,8,9,9,92,023,023,0$
$, 2,3,8,0,9,923,023,023,0,1, I$
$, 423,923,023,023,0,1, I \quad, I$

, 8 ,
25
, I ,
$, 4,5 \quad, 6$
$, 7,8$

| $94,024,024,0$ |
| ---: | :--- | ---: | ---: | ---: |

26,0
$, 2,2 \quad, 2$



| , 5 | , 1 | , 1 | , 1 | , 2 | , 2 | , 2 | , 2 | , 2 |
| ---: | ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- |
| , 7 | , 2 | , 2 | , 3 |  |  |  |  |  |
| , 2 | , 2 | , 3 | 0 | , 3 | , 4 | , 4 | , 4 | , 4 |

A TABLE for finding the Mean Diameters of Rum

$, 2,3,8 \quad, 9 \quad, 9 \quad, 924,024,024,0$
, 4 , $9,24,024,024,0$, 1,1 ,

 $, 925,0,025,0,1 \quad, 1 \quad, 2$
$27,0,1 \quad, 125,110,1$
 ,7 7

Puncleons or Cafks of the Firgt Form


$$
\begin{array}{cc|cc|c|c|c|}
1 & , 2 & , 4 & , 5 & , 5 & , 5 & , 6 \\
& , 3 & , 5 & , 6 & , 6 & , 6 & , 7 \\
\hline 4 & , 5 & , 6 & , 7 & , 7 & , 7 & , 8 \\
& , 6 & , 7 & , 8 & , 8 & , 8 & , 9
\end{array}
$$


 $, 2,3 \quad 8 \quad, 9 \quad, 9 \quad, 925,025,025,0,1,1$, $, 4 \quad 9.25,025 ; 025,0,1 \quad, 1 \quad, 1 \quad, 2,2$
$, 5,625,3 \quad, 1 \quad, 1 \quad, 11 \quad, 2 \quad, 2 \quad, 2 \quad, 31,31$,
$\begin{array}{lllllllllll}, 7 & , 1 & , 2 & , 2 & , 2 & , 3 & , 3 & , 3 & , 4 & , 4 & , 4 \\ , 9 & , \frac{2}{3} & { }_{3}^{2} & , 3 & , 3 & , 4 & 4 & , 4 & , 5 & , 5 & , ~\end{array}$
27
, I , , 2 ,
$, 4,5$

| , 6 | , 8 |
| :---: | :---: |
| , 8 |  |

$, 926,00_{25}, 026,0$
$\begin{array}{rrrr}28,0 & , 1 & , 1 & , 1 \\ , 3 & , 4 & , 2 & , 2 \\ , 6 & , 7 & , 4 & , \\ , 5 & , 6 \\ , 9 & 29 & , 7 \\ 29 & , 1 & , 8 \\ , 2 & , 3 & , 0 & ,\end{array}$



A TABLE for finding the Mean Diameters of Rum


## Puncheons or Cafks of the Firft Form.

$\overline{26}, 0, \frac{24,0}{25,3}\left|\frac{, 1}{25,4}\right| \frac{, 2}{25,4}-\frac{, 3}{25,4}\left|\frac{, 4}{25,5}\right| \frac{, 5}{25,5}|-6|-\frac{, 7}{25,5}\left|\frac{, 8}{25,6}\right|-\frac{, 9}{25,6}$
 $, 7,8-8$, 9 ,9 ,926,0,26,026,0 $, 1, I$
 $, 2,3 \quad, 8 \quad 9 \quad, 9 \quad, 927,027,027,0 \quad, 1 \quad, 11$
 ,5,627,
, 8 , 9
29

,4,5, 7



$, 2,2 \quad, 2 \quad, 2 \quad, 3 \quad 3 \quad, 3 \quad, 4 \quad, 4 \quad, 4 \quad, 5$




## Puncheons or Cafis of the Firft Forme

$\overline{28 \quad 3}\left|\frac{26,0}{27}, \frac{, 1}{27,6}\right|-\frac{2}{27,6}-\frac{, 3}{27,6}\left|-\frac{4}{27}\right|-\frac{, 5}{27,7}\left|-\frac{, 6}{27,7}\right|-\frac{7}{27,8}\left|\frac{, 8}{27,8}\right| \frac{, 9}{27}, 8$ $\begin{array}{lllllllll} & 4,5 & , 6 & , 7 & , 7 & , 7 & , 8 & , 8 & , 8 \\ , 9 & , 9\end{array}$


$\begin{array}{llll}, 2 & , 1 & , 2 & , 2 \\ , 2 & , 3 & , 3\end{array}$
. 3
,6,
,
$30^{\circ}$

| , 1 | , 7 | , 7 | , 7 |
| :--- | :--- | :--- | :--- | :--- |
| , 2 | , 8 | , 9 | , |

,4 , 9,29,029,029,0
,5,629,0, 1
31 ,
, 1 , 2
, 4

$, 930,030,030,0,1 \quad, 130,1$
32,0
,3
, 6 ,
933
$33 \quad 1$
,2 ,3 ,8
,5,6 1
,8
$\begin{array}{r}1134 \quad, 9 \\ \hline 13\end{array}$
A TABLE for funding the Mean Diameters of Runt




## A TABLE for finding the Mean Diameters of Rum


 64 , $31,031,031,0,1,1,10_{2}, 2$ ,5,6 31,0
 33 , 1

| , 3 | , 6 | , 6 | , 6 |
| ---: | ---: | ---: | ---: |
| , 5 | , 7 | , 7 | 9 |
| , 6 | , 8 | , 8 | , 8 |

$, 7,8 \quad, 9 \quad 9 \quad 9,32,032,032,0,1 \quad, 1$

 | , 9 | 35 | , 7 | , 7 | , 7 | , 8 | , 8 | , 8 | , 9 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| , 8 | 9 | , 33 |  |  |  |  |  |  |  | $, 2,3 \mid, 9 \quad 9 \quad, 93,30,33,0 \quad 11 \quad 1 \quad 1 \quad, 2$



Puncheons or Cafks of the Firf Form


4 TABLE for finding the Mean Diameters of Rum


 37
, 2,3 , $9 \quad, 9 \quad 9$ $, 435,035,035,0$ , 7 , 2
,8,9 $3^{8}$, 1,
, 4,
, 5
, 6
, 5
,
, 24,5

| 4,5 | , 7 | -7 | ,-7 |
| ---: | :---: | :---: | :---: |
| , 6 | , 8 | , 8 | , 8 | めッ



 an N

7
, 2
, 3
, 4
, 5
, 6
, 7
, 8
-9
66

, 0 35, 35,1 | I | I |
| :--- | :--- |

Puncheons or Calks of the Firft Form.


$2,3,8 \quad, 9 \quad, 9 \quad, 934: 034,034,0,1 \quad, 1$ $, 4,934,034,0,34,0 \quad, 1 \quad, 1 \quad, 1 \quad, 2 \quad, 2,2$


## Puncheons or Calks of the Firft Form.





## Puncheons or Cafks of the Fi,yt Fornt



A TABLE for finding the Mean Diameters of


Wine Pipes or Cafks of the Second Form.
$\left.\overline{18, \quad, 0}\left|\frac{16}{17,0}\right|-\frac{1}{17,3}\left|\frac{, 2}{17,4}\right| \frac{, 3}{17: 4}\left|-\frac{4}{17,4}\right|-\frac{5}{17,5}\left|\frac{, 6}{17,5}\right| \frac{, 7}{17,5} \frac{, 8}{17,6} \right\rvert\, \frac{, 9}{17,6}$

| , 1 | , 2 | , 4 | , 4 | , 4 | , 5 | , 5 | , 6 | , 6 | , 6 | , 71 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| , 3 | , 5 | , 5 | , 5 | , 6 | , 6 | , 7 | , 7 | , 7 | , 8 | , 8 |
| , 4 | , 5 | , 6 | , 6 | , 6 | , 7 | , 7 | ,$\frac{7}{8}$ | , 8 | , 8 | , 9 |
| , 9 |  |  |  |  |  |  |  |  |  |  |


$9, \quad, 9,9,918,018,018,0,1,1,1,2$

,2 $18,0,1 \quad$,
$\begin{array}{rrrr}3 & , 4 & , 1 & , 2 \\ , 5 & , 2 & , 3 & , 3\end{array}$

| , 6 | , 7 | , 3 | , 4 | , 4 |
| ---: | ---: | ---: | ---: | ---: |
|  | , 8 | , 4 | , 5 | , 5 |
| , 9 | 20 | , 5 | , 6 | , 7 |
| 20 | , 1 | , 6 | , 7 | , 7 |
| , 2 | 3 | , 7 | , 8 | , 8 |
| , 4 | , 8 | , 9 | , 9 |  |
| , 5 | , 6 | $, 9 \frac{18}{18}, 9$ | 19,0 |  |

$17 \mathrm{I}_{19,019,0}^{19,019} 11^{19}$
, $8,9 \quad 1 \quad, 1$
21
I, ,
,4
, 7
22, 0,1
,2 20,020,0 2c,020,0

 ,7) $921,02_{21,021,0}, 1|, 1|, 1|, 2|, 3$

A I' ABLE for finding the Mean Diameters of

 $25,5 \quad 9 \frac{1}{\frac{1}{2}} \frac{0}{\circ}, \frac{8}{0} 20,0,20,0,1 \quad, 1 \quad, 1 \quad, 2,2,2$





$20 \quad, 119,319,319,419,419,419,519,519,5,19,6119,6$
1, ,2 $, 4 \quad, 4 \left\lvert\, \begin{array}{llllll}5 & , 5 & , 5 & , 6 & , 6 & , 6 \\ 5 & , 7\end{array}\right.$

| , 3 | , 5 | , 5 | , 5 | , 6 | , 6 | , 7 | , 7 | , 7 | , 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , 8 |  |  |  |  |  |  |  |  |  |
| -5 | , 6 | , 6 | , 6 | , 7 | , 7 | ,$\frac{7}{3}$ | , 8 | , 8 | , 9 |
| , 9 |  |  |  |  |  |  |  |  |  |
| , 6 | , 7 | , 7 | , 7 | , 8 | , 8 | , 8 | , 9 | , 9 | 20,0 |

$, 7,8$, 8 , 8 , 8 , $, 9,0,20,020,0$, $\frac{0}{3}$, I
$: 9,9,9,920,020,020,0,1,1,1,2$


,3,4
,5 ,
, 6,7
,9 22
22
$\begin{array}{lll}, 2 & , 3 & , 7 \\ , 4 & , 8\end{array}$
${ }^{2} 2$

| $\quad 3$ |
| :--- |
| , 4 |
| , 5 |
| , 6 |
| , 7 |
| , 8 |

3
, 4
, 6
$7121,0^{\frac{20}{2} 2^{2}}, \frac{2}{9}$
${ }_{23}, 8$,
,: ,
, 1 ,

,


| 4 |
| :--- |
| 4 |

$\frac{8}{2} 21,0$


A TABLE for finding the Meas Diameters of


| 1 | , 4 |
| :--- | :--- |
| , 5 | , 5 |
| , 6 | , 6 |
| , 7 | , 7 |
| , 8 | , 8 |
| 9 | , |

22,0,1
,3,4
, 5,5

923 23
,2,
$, 5,6$, $9 \frac{2}{2} \frac{4}{2}, \frac{9}{0} 22$, $, 722^{\prime} 0_{22}, 011^{22,}$

| 18,9 | , | , 1 |
| ---: | ---: | ---: |
| 24 | , 0 | , 2 |
| $, 1,2$ | , 3 | , 3 |
| , 3 | , 4 | , 3 |
| $, 4,5$ | , 5 | , 5 |
| , 6 | , | , 6 |
| $, 7,8$ | 7 | , 7 |
| , 9 | , | , 8 |
| $25,0,1$ | , 9 | , 9 |

$$
, 2{ }_{23}, 923,0
$$

$$
\text { , } 3
$$

, 6
$26^{9}$

$\left.\overline{22, \quad, 0}|21,3|-\frac{1}{21,3}\left|-\frac{2}{21,4}\right|-\frac{3}{21,4}|-4|-\frac{, 5}{21,4}\left|\frac{, 6}{21}\right| \frac{, 7}{21,5}-\frac{8}{21,5} \right\rvert\, \frac{, 9}{21,6}$


Pipes or Cafes of the Second Form.
$\overline{24,0}\left|\frac{22,0}{23,3}\right| \frac{, 1}{23,3}\left|\frac{, 2}{23,4}-\frac{, 3}{23,4}\right| \frac{, 41}{23,4}, 5, \frac{, 6}{23}, \frac{, 7}{23,5}, 8 \left\lvert\,-\frac{, 9}{23,6 \mid 23,6}\right.$

$\begin{array}{lllllllllll}, 3 & , 5 & , 5 & , 5 & , 6 & , 6 & , 7 & , 7 & , 7 & , 8 & , 8 \\ , 5 & , 6 & , 6 & , 6 & , 7 & , 7 & , 7 \\ 8 & , 8 & , 8 & , 9 & , 9\end{array}$ ,6 ,7 ,7 ,7 , 8 , 8 ,8 ,9 $, 924,0,24,0$
$\rightarrow 7,8$
,9 ,9 ,9 , $924,024,024,0,1,1,1 \quad, 2$
$25,0, \frac{23}{24}, \frac{0}{0} 24,024,0 \quad, \quad, 1 \quad, 1 \quad, 2,2,2$ $, 2,24^{\prime} 0,1,1 \quad, 2,2,2,3,3,3,4$
,3
,6,7

26
,2,3 $\quad, 7$
 $, 725,025,0 \quad 1 \quad, 1 \quad, 2,2,2,3,3$

$, 7,8 \quad, 7 \quad, 7 \quad, \frac{7}{8} \quad, 7 \quad, 8 \quad, 9 \quad, 926,026,026,0$



A TABLE for finding the Mean Diameters of




## Wine Pipes or Calks of the Second Form.




| $, 7,8$ | , 7 | , 7 | , 7 | , 8 | , 8 | , 8 | , 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| , 8 | , 9 | , 0 | 26,0 |  |  |  |  | $\begin{array}{lllllll}97,9 & , 9 & , 9 & 26,026,026,0 & , 1 & , 1 & , 1\end{array}$



, 8 , $, 7=7,027,0$

I, '
$30,0,1$,y .9 ,9 ,928, ,228,028,028,028,0

|  <br>  <br>  |
| :---: |

Wine Pipes or Cafks of the Second Form.
$\overline{28,4,5} 27,627,6 \overline{27}, 627,727,727, \frac{7}{8} 27,8, \overline{27}, 8,27,9127,9$

$9,9,9,928,028,028,0,1,1,1,1$
$29,0,1 \frac{2}{2} \frac{7}{8}, \frac{2}{0} 28,0,28,0,1 \quad, 1 \mid, 1 \quad, 2,2,2,3$ $, 228,0,1 \quad, 1,2,2,2,3,3,3,4$
,3
,5
,6,7
-, 8
,930
30
,2 ,3 ,
$\begin{array}{llllll}1,7 & , 8 & , 8 & , 7 & , 8 & , 8 \\ , 9 & , 8 & , 9 & , 9 & , 929,029,029,\end{array}$
$, 5,6,9 \frac{28}{29}, \frac{9}{0} 29,029,0,1,1,1,2$
,7,29,029,0 ,1 ,1 ,2, ,2

$, 7,8$,7
$32,0,1), 9$
$, 230,030,0$
,3



A TABLE for finding the Mean Diameters of


Wine Pipes or Cafks of the Second Form.

$34,0,1 \quad, 9 \quad, 0 \quad, 9 \quad, 3 z, C$, $\frac{0}{1}, 1 \quad, 1 \quad y_{2} \quad, 2$


A TABLE for finding the Mean Dianseters of


Wine Pipes or Cafks of the Second Form.


A TABLE. for finding the Mear Diameters of



Wine Piper or Caftes of the Second Form.
























 $-\frac{16,0}{10}\left|\frac{16,1}{87}\right| \frac{, 2}{87}\left|\frac{, 3}{9}\right| \frac{, 4}{9}\left|\frac{, 5}{92}\right| \frac{, 6}{9}\left|\frac{, 7}{94}\right| \frac{, 8}{95} \frac{, 9}{96}$














 $222+2244 \cdot 46|25025 \quad 3| 25 \quad 525 \quad 7|26 \quad 1| 26 \quad 3 \mid 26$ $2325 \quad 3.2 ; \quad 5125 \quad 7126 \quad 2: 264,266270,27 \quad 327527$
 $2527427 \quad 7: 31 \mid 284286291293129630030$





 $32,352_{2} 35 \quad 5.360 \mid 36$ $33 \left\lvert\, \begin{array}{lllllllllllllllll}50 & 3 & 3 & 6 & 6,37 & 1 & 37 & 5 & 38 & 0 & 38 & 2 & 39 & 7 & 39 & 2 & 39 \\ 5 & 40 & 1\end{array}\right., 807$ $34374377382 / 386391 / 39440040$ 3/40741 $2 \mid .910$




 | 27 | 33 | 1 | 33 | 4 | 33 | 7 | 34 | 2 | 34 | 4 | 34 | 7 | 35 | 2 | 35 | 5 | 36 | 0 | 36 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 28 | 34 | 3 | 34 | 6 | 35 | 1 | 35 | 4 | 35 | 7 | 36 | 2 | 36 | 5 | 37 | 0 | 37 | 3 | 37 | $29,35 \quad 5 \mid 360,36$

 3 I






in Wine Gallons ant Pints.


in Wine Gallons and Pints.




 | 34,66 | 5 | 67 | 1 | 67 | 668 | 2 | 68 | 7 | 69 | 3 | 70 | 0 | 70 | 4 | 71 | 1 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |




 40,785170
 $42: 82 \quad 2,83 \quad 0,83584$ $43,842,85 \quad 0,85 \quad 5,86$ $4486 \quad 1,87087 \quad 5,88 \quad 3189$ I $896690491 \quad 292 \quad 092 \quad 6,917$

 $30636,642,646,65 \quad 265 \quad 666$


 $34,722,72 \quad 773$ 374.074 $\begin{array}{lllllllllllllllll}35,74 & 3 & 75 & 0 & 75 & 5 & 76 & 1 & 76 & 6 & 77 & 3 / 78 & 0,78 & 5 & 79 & 2 & 79 \\ 7\end{array}$
 $377857792,79780 \quad 581 \quad 181 \quad 6824831 / 836843$

 $4085085 \quad 58638870,876,88 \quad 318918997,9049121,305$







A TABLE of the Conterits of Cylinders



A TABLE of the Contents of Cylinders

in Wine Gallons and Pints.

|  |  |  |  |  |  |  | ,6 |  |  |  | \| |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 347 | 35 | $35^{2}$ | 35 | 356 |  | 3 | $3{ }^{6} 3$ | 365 |  |  |
|  | 125 | 26 | 7 | 127 | 284 | 4129 | 13011 | 1307 |  | 324 | 4 |
|  | 28 |  | 3 |  | - | 32 | 33 | 344 | $35 \quad 3$ | 36 I |  |
| 3 | 32 | 331 | 340 |  | 355 | 364 | 37 | 381 | 390 | 39 |  |
|  | 35 | 365 | 37 |  | 392 | 40 | 40 | 416 | 425 | 434 |  |
|  | 392 | 401 | 410 | 417 | 426 | 6435 |  | 453 | 462 | 472 |  |
|  | 426 | 43 |  | 45 3 | 463 | 3472 | 48 | 491 | 50 | 507 |  |
|  | 469 | 470 | 48 | 490 | 497 | 7510 | 516 | 526 | 53 | 545 |  |
|  | 496 | 505 | 515 | 523 | 534 | 4543 | 553 | 563 | 57 | 582 |  |
|  | 53 | 541 | 551 | 56 I | 570 | 580 | 59 | 60 o | 610 | 617 |  |
|  | 565 | 575 | 585 | 59 | 605 | 615 | 625 | 635 | 645 | 655 |  |
|  | 601 | 611 | 62 | 632 |  | 652 | 662 | 672 | 682 |  |  |
|  |  | 64 | 656 | 665 | 67 | 686 | 697 | 707 | 720 |  |  |
|  | 67 1 | 681 | 692 | 702 | 71 | 723 | 734 | 744 | 755 | 76 |  |
|  | 20 5 | 715 | 726 | 736 | 75 | 760 | 771 | 781 | 791 | 80 |  |
|  |  | 751 | 76 | 773 | 78 |  | 805 | 815 | 82 |  |  |
|  |  | 785 | 796 | 807 | 82 | 831 | 842 | 853 | 864 |  |  |
|  | 8 | 82 I | 83 | 844 |  | 865 | 877 | 890 | 90 |  |  |
|  | 8 | 855 | 8 | 88 | 8 | 903 | 914 | 925 | $93 \%$ | 95 |  |




in Wine Gallons and Pints.



For reducing Rum Puncheons, or Cafks of the Firft Form, io Cylinders.

|  | , 0 | , 11 | ,2 | ,3 | , 4 | , 5 | , 6 | , 7 | , 8 | , 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | , 7 | ,7 | , 8 | ,9 | ,9 | 1,0 | 1, I | I, 1 | 1,2 | 1,3 |
| 2 | 1,3 | 1,4 | 1,5 | 1,5 | 1,6 | 1,7 | 1,7 | 1,8 | 1,9 | 1,9 |
| 3 | 2,0 | 2, I | 2,1 | 2, 2 | 2,3 | 2,3 | 2,4 | 2,5 | 2,5 | 2,6 |
| 4 | 2,7 | 2,7 | 2,8 | 2,9 | 2,9 | 3,0 | 3, 1 | 3,1 | 3,2 | 3,3 |
| 5 | 3,4 | 3,4 | 3,5 | 3,6 | 3,6 | 3,7 | 3,8 | 3, 8 | 3,9 | 4,0 |
| 6 | 4,0 | 4, 1 | 4,2 | 4,2 | 4,3 | 4,4 | 4,4 | 4,4 | 4,6 | 4,6 |
| 7 | 4,7 | 4,8 | 4,8 | 4,9. | 5,0 | 5,0 | 5, I | 5,2 | 5,2 | 5,3 |
| 3 | 5,4 | 5,5 | 5.4 | 5,6 | 5,6 | 5,7 | 5,8 | 5,8 | 5,9. | 6,0 |
| 9 | 6,0 | 6, 1 | 6,2 | 6,2 | 6,3 | 6,3 | 6,4 | 6,5 |  | 6,6 |

TA B L E II.
For reducing Wine Pipes, or Cafks of the Second Form, to Cy!inders.

|  | , 01 | , 1 | ,2 | , 3 |  |  | , |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ,7 |  | , 8 | ,9 | 1,0 | 1,0 | 1, 1 | 1,2 | I, |
| 2 | 1,3 | 1,3 | 1,4 | i, ${ }^{\text {¢ }}$ | 1,5 | 1,6 | 1,7 | 1,7 | 1, 8 | 1,9 |
| 3 | 1,9 | 2,0 | 2,0 | 2, I | 2,2 | 2,2. | 2,3 | 2,4 | 2,4 | 2,5 |
| 4 | 2,6 | 2,6 | 2,7 | 2,8 | 2,8 | 2,9 | 2,9 | 3,0 | 3, I | 3,1 |
| 5 | 3,2 | $3 \cdot 3$ | 3,3 | 3,7 | 3,5 | 3,5 | 3,6 | 3,6 | 3,7 | 3,8 |
| 6 | 3,8 | 3,9 | 4,0 | 4,0 | 4, I | 4,2 | 4,2 | 4,3 | 4,4 | 4,4 |
| 7 | 4,5 | 4,6 | 4,6 | 4,7 | 4,8 | 4, 8 | 4,9 | 4.9 | 5,0 | 5,1 |
| 8 | 5, I | 5,2 | 5,2 | 5,3 | 5,4 | 5,4 | 5,5 | 5,5 | 5,6 | 5,7 |
| , | 5,8 | 5,8 | 5,9 | 6, c | 6,0 | 6,1 | 6, 1 | 6,2 | 6,3 | 6,3 |

For reducing Cafks of the Third Form to Cylinders.

|  | , 01 | , i | ; 21 | , 31 | , 41 | , 5 | ,61 | , 7 | ; 3 | ,9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | ,6 | ,7 | ,7 | ,8 | , 8 | , $)$ | 1,0 | 1,0 | 1, I | 1,1 |
|  | 1,2 | 1,3 | 1,3 | 1, ${ }^{\text {a }}$ | 1,4 | 1,5 | 1,6 | 1,6 | I,7 | 1,7 |
| 3 | 1,8 | 1,9 | I,9 | 2,0 | 2,0 | 2, I | 2,2 | 2,2 | 2,3 | 2,3 |
|  | 2,4 | 2,5 | 2,5 | 2,6 | 2,6 | 2,7 | 2,8 | 2,8 | 2,9 | 2,9 |
| 5 | 3,0 | 3, 1 | 3, 1 | 3,2 | 3,2 | 3,3 | 3,4 | 3,4 | 3:5 | 3,5 |
| 6 | 3,6 | 3,7 | 3,7 | 3,8 | 3,8 | 3,9 | 4,0 | 4,0 | 4, I | 4,1 |
| 7 | 4,2 | $4 \cdot 3$ | 4,3 | 4,4 | 4,0 | 4.5 | 4,6 | 4,6 | 4,7 | 4,7 |
| 8 | 4,8 | 4,9 | 4,9 | 5,0 | 5,4 | 5.1 | 5,2 | 5,2 | 5,3 | 5.3 |
| 9 | 5,4 | 5,5 | $5 \cdot 5$ | 5,6 | 5,6 | 5,7 | 5,8 | 5,8 | 5,9 | 5,9 |





Explanation and UJe of the Tables for finding the Mean Diameters of Cafks of the Firft, Second, and Third Forms.

THE two firft fet of Tables will be found to comprehend almoft all cafks which may be met with in the practice of gauging ; for fmall ones under the dimenfions of thefe tables, are fcarce ever gauged; but that nothing fhould be wanting to extend them generally, there are alfo added three fmail tables to be ufed when the bung and head diameters of the cafk are too big or too little for them, which perhaps will not happen once in five hundred times.

To the firt Table, cafks that are much bilged at the bung belong: Rum puncheons are generally of this form ; and wine pipes, being lefs bilged at the bung, belong to the fecond. Yet there are fume rum puncheons that belong to the fecond form, but perhaps none to the third. A little experience will foon enable the gauger to judge precifely of the variety of a cafk. When the ftaves are much curved, that is, appear circular between the bung and head, then that cafk is of the Firft Form; but if the ftaves between the bung and head are not much curved, that is, tolerably ftraight, then that cafk is of the Second Form ;
and when they appear almof entirely fraight from the bung to the head, like two flraight ftaved veffels joined or butted together, then that cank is of the 1 hird Form : There are, however, but few cafks belonging to this form ; and hence it was deemed unnecefiary to have a large table for finding their mean diameters; as the fmall table with very little additional trouble, will anfwer the purpofe.

## Ruile.

To find the mean diameter of a cafk from the firft and fecond tables. Find the head diameter at the top, and bung diameter at the fide, in the angle of meeting is the mean diameter required.

> Examples.

Suppofe the bead and bung diameters of a rum puncheon or cafk of the firft form to be as follow; required the mean diameter?

```
Head Diam. 28,6
Bung Diam. 32,7
```

Having found 28,6 at the top, and 32,7 at the fide, in the angle of meeting is 31,3 for the mean diameter required.
N. B. When two figures of tenths fand at the fide, the mean diameter is the fame for both; except when two fmall figures are placed the one above the other like a fraction, then the upper figure is the tenths of the mean diameter to the leffer bung diameter, and the lower figure belongs to the greater. Thus,

Suppofe the head diameter 20,2 , and bung diameter
ameter 25,1 or 25,2 ; then the mean diameter is 23,5 for both. But fuppofe the head diameter 20,3 , and the bung diameters 25,1 and 25,2 as before; then the mean diameter for the former is 23,5 , and for the latter 23,6 .

UJe of the Table of the Contents of Cylinders, ins Gauging Cafis, \&c.

## 1 Rule.

Find the mean diameter of the cafk at the top, and the length, omitting the tenths, at the fide; then in the angle of meeting is the content in wine gallons and pints, for the whole inches in length; and to this add the gallons and pints in the margin, anfwering to the odd tenths in length, and you have the true content of the cafk.

> Exampie.

Suppofe the head and bung diameters, and length of the cafk of $\Rightarrow$ firft form to be as follow ; required the content in wine gallons?

$$
\begin{aligned}
& \text { Head Diam. } 28,3 \\
& \text { Bung Diam. } \\
& \text { Length, }
\end{aligned} \quad \begin{aligned}
& 31,7
\end{aligned}
$$

Having found the mean diameter, in the firft table to be 31,9 ; then with this mean diameter enter the table of cylinders, and oppofite to $4!$ on the fide, we have the content for the whole inches in length, gal. pts.

$$
1417
$$

for, 7 add 23 found in the margin.
1442 content of cafk, required.

More examples for illuftrating the ufe of thefe tables would be unneceffary; and it may be obferved, that their utility in gauging is as extenfive as their operation is fimple and expeditious.
$N . B$. Left the length of the cafk thould be too great or too little for thefe tables; there is given the content at the head line, for 10 inches in length, which is to be ufed as in this example.

Suppofe the mean diam. 31,9$\}$ required the $/$ Length, $34,2\}$ content.
Here the length being lefs than in the table, I add 10 to it, and find the content for 44,2 as before directed, which is

Subtract for 10 inches
gal. pts.
1527
$34 \quad 4$
1183 content req.
If the length be too great let 10 inches be fubtracted, and to the content anfwering that length, add that for 10 inches, and the fum will be the whole content.

Ufe of the three fmall Tables, in finding the Mean Diameters of Cafis.

## Ruic.

Subtract the head diameter, from the bung dimeter, and with the difference enter the proper table belonging to the cafk, and finding the inches at the fide and tenths at the top, in the angle of meeting is a number which being added to the head diameter gives the mean diameter required.

Example.

Cung Example.
Suppofe the diam. 35,6 of a calk of the head biam. 26,9 $\}$ firft form.

## Difference, 8,7

With 8,7 having entered the Table No. I, I find 5,8 ; then

To 26,9
Add 5,8
Gives 32,7 the mean diameter required.

> Remark.

Thefe three tables are of more general ufe than the two firlt, which find the mean diameter at once, and by thefe the firlt two were formed; fo that only with the additional trouble of fubtracing and adding, thefe tables fupply the place of the others; and in cafe of any doubt of a typographical error being in the firt two large tables, let thefe be ufed, and the miftake, if any, may be difcovered and rectified.

## Of Ullaging of Casks.

This important part of gauging has generally been confined to the flide-rule; by the uncertainty of which, many palpable loffes have been fultained, by the national revenue, as well as by the buyer and feller alternately. No inftrument ought to be the criterion or judge of property, the divifions of which are fometimes fo clofe, that the coincidence of feveral are to be gueffed at ; and there are many inftances wherein a fmall error of the inftrument, will caufe feveral gallons of a miftake; and fuch errors pertain to the largeft and beft executed flide-rule that can be made. The unavoidable errors of the inftruments for taking the dimenfions, are as much as hould be admitted; but an accumulation of them is certainly derogatory to fcience, and at the fame time injurious to trade and fair dealing, reducing them to a fpecies of lottery; as no one knows whether the feller or the buyer may not gain in a fingle pipe of Madeira wine, feveral gallons the one from the other ; fo that a transfer of property to the amount of fome pounds, may, and really generally does, attend a fingle operation of this erronious inftrument; and this, too, without any crime in the gauger; for with all his care, the rule may err feveral gallons from the truth. Calculation, therefore, as being perfectly true in itfelf, ought by all means to be practifed in gauging in general, but particularly in ullaging. Hence the following method, although attended with a divifion and multiplication in each
operation, is confequently to be preferred; and dealers in wine, fpirits, $E^{\circ} c$. ought not to depend on any other.

Explanation and Ufe of the Table for finding the Uillage of Cafks.

Thefe tables find the quantity drawn out of a cafk if it be more than half full ; but if the cafk be lefs than half full, they find the liquor remaining in it.

To find the quantity drawn out of a cafk more than half full, whofe axis lie parallel to the horizon ; that is, a cafk lying on its fide.
Rule.

Let the mean diameter, and thence the content of the cafk be found: Then from the dry inches fubtract half the difference between the bung and mean diameter, and divide the remainder with three cyphers annexed to it, by the mean diameter, and the quotient is a verfed-fine, which being found in the table, the number ftanding againft it is the quantity drawn out of the cafk, fuppofing it to contain 1000 gallons: Hence if the whole content of the calk be multiplied by this number, and divided by 1000 you have the exact quantity drawn thereout.

Note, The two decimal figures may be rejected; obferving if they be not more than, 50 to increafe the number of gallons by one; but if lefs, they are altogether to be neglected.

Gallons. Gallons.
 Example.
Suppose the head and bung diameters, length and dry inches, to be as follow; required the quantity drawn out of it, the cark being of the firlt form? Head diam. 24,0

Bung diam. 32,0
Dry inches, 14,0
Length, 48,0
The mean diameter is 29,4 ; and content 141 gall.

From 32,0
Take 29,4
Diff. 2,6 Rem. 12,7

Half Diff. i, 3
$29,4) 12,7000(432$ verfed-fine
againft which is 413,69 , or 414 ; then
Multiply 4I 4
By
141

> 414
> 1656
> 414

581374 drawn out. That is the gal. pt.
calk wants $5^{8} 3$ of being full; and if this be taken

120 Explanation of the Preceding Tables.
taken from the whole content, the remainder will be the liquor ftill remaining.

Remark. If the cafk be lefs than half full ; ufe the wet inches, and you will (proceeding according to the rule) have the quantity remaining in the calk.

> THE END.


In the Prefs, and Speedily will be Publi/bed,
An Edition of Gough's Arithmetic. Corrected and fitted for American Schools, by the Author this Work.
$1+2$
1)
?


